Anisotropic Two-Dimensional Scattering Part II: Finite Depth and Refractive Index Effects

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Introduction

COMPREHENSIVE surveys of experimental activity concerning the scattering of light are present in literature reviews.[1-5] The information presented here is an extension of the work reported in Refs. 4 and 5. Latex paint was used as scattering centers in Ref. 4, while spherical polystyrene particles of fixed diameters were used in Ref. 5. In Ref. 4, the effects of a finite depth on multiple scattering were presented. The agreement between the theoretical and measured scattered intensities was fair in magnitude, shape, and position. Multiple scattering from a semi-infinite medium was the subject of Ref. 5. An important finding of this study was that the intensity distribution curves for different particle sizes collapse to a single curve by using (1-*g*) as a correlation parameter with the optical thickness (*g* is the asymmetry factor).

In the present experimental investigation, a helium-neon laser beam was incident normal to the surface of a finite depth, scattering medium. The radiant energy emerging normally from the liquid surface was measured as a function of the radial distance from the laser beam R. The scattering centers used were high-quality spherical polystyrene particles (diameter = 0.261 μm). These particles were immersed in liquids (double-distilled water, $n = 1.331$, or high-purity ethylene glycol, $n = 1.472$) to achieve various concentrations. The magnitude of these concentrations was determined by imposing optical depth requirements.

In order to model this physical situation, the following assumptions were made: 1) steady-state laser beam and medium conditions existed during the time required to record the data; 2) the detected radiation was a function of the variables R and Z (the physical depth); 3) the medium was homogeneous and, in the R direction, symmetric and very large while it was of finite depth; 4) the bottom was black; 5) multiple, anisotropic scattering occurred with no emission; and 6) spheres were pure scatterers and the liquids exhibited only slight absorption.

Under these conditions, experimental data were acquired with the following purposes in mind: 1) to demonstrate that the anisotropic intensity distribution curves will collapse to an effective isotropic situation, for a given depth (τ_0^*), by using (1-*g*) as a correlation parameter and 2) to perceive the effect of the refractive index of the liquids on the data received.

Background

Implementation of assumption 6 requires the optical thickness τ to be defined as

$$\tau = \sigma L \tag{1}$$

where L is the radius or the depth of the medium and σ the scattering coefficient. If the effective scattering cross section is $\bar{C}$ and N is the particle number density, then

$$\sigma = N\bar{C} \tag{2}$$

So

$$\tau = N\bar{C}L = NV_p(\bar{C}/V_p)L \tag{3}$$

where V_p is the volume of a particle. And if

$$C = \bar{C}/V_p \tag{4}$$

then

$$\tau = NV_pCL \tag{5}$$

Finally if $\bar{n}$ is the number of particles,

$$N = \bar{n}/(\bar{V} + \bar{n}V_p) \tag{6}$$

where $\bar{V}$ is the volume of the liquid present. Then

$$\tau = \eta CL \tag{7}$$

Note that η is the total volume of the scattering particles divided by the total volume (liquid plus particles). With the exception of C, all quantities on the right side of Eq. (7) are measurable. C was determined directly from the Mie theory.

The absorption of the liquid was included in the overall optical thickness,

$$\tau_{(\text{total})} = \tau + \tau_{(\text{liq})} \tag{8}$$

where $\tau_{(\text{liq})}$ represents the contribution to the optical thickness due to the absorption of the liquid. Thus, error is introduced since there is scattering by the liquid and absorption by the particles. This error is small and the resulting single scattering albedo is only slightly less than 1 (≈ 0.99).

Laboratory Effort

The laboratory apparatus was similar to that used in Refs. 1-5; a few differences did exist in order to conform to the assumptions. First, an all-glass cylindrical tank contained the scattering medium. Its diameter and maximum depth was 26.5 and 30 cm, respectively. Second, to simulate a finite optical depth, the tank was equipped with a movable bottom plate whose top surface was coated with 3M® Black Velvet paint. The detector probe used was the same as that in Refs. 2-5 (acceptance angle = 2.750 deg).

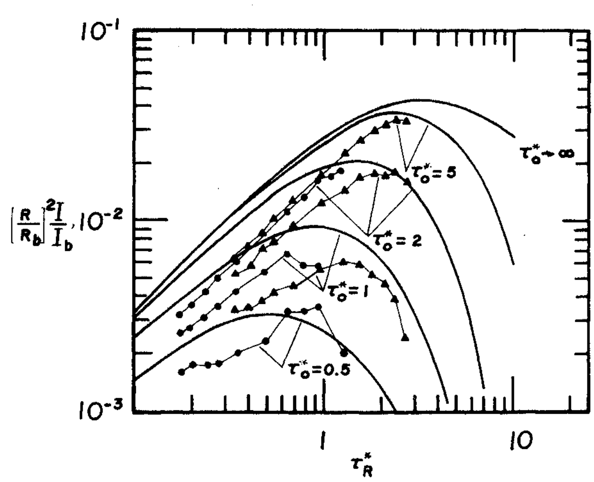

Fig. 1 Comparison of backscattered experimental (symbols) and theoretical nondimensional intensities vs radial optical thickness for a double-distilled water medium with a black bottom plate and 0.6328 μm incident light (● 0.5 ml and ▲ 1.0 ml of spherical particles).

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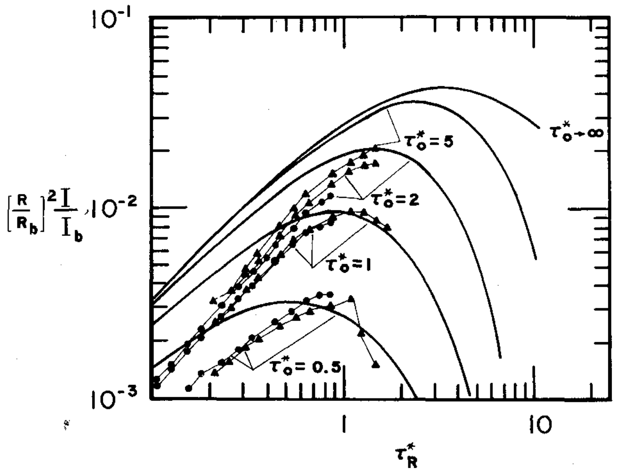

Fig. 2 Comparison of backscattered experimental (symbols) and theoretical nondimensional intensities vs radial optical thickness for an ethylene glycol medium with a black bottom plate and 0.6328 μm incident light (● 2.0 ml and ▲ 4.0 ml of spherical particles).

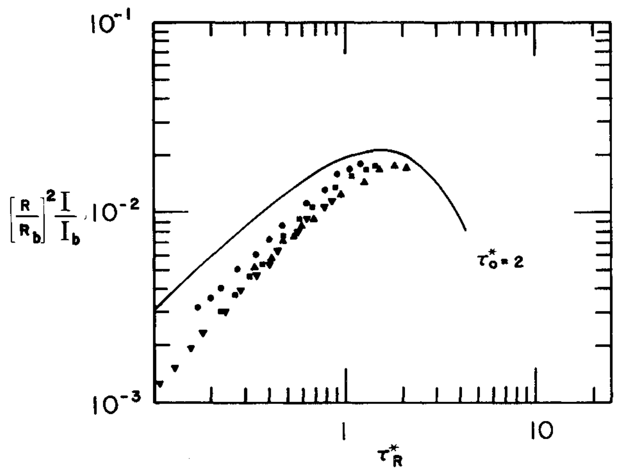

Fig. 3 Comparison of backscattered experimental (symbols) and theoretical nondimensional intensities vs radial optical thickness in the case of an optical depth of 2, a black bottom plate, and 0.6328 μm incident light (particle volumes: ● 0.5 ml in water, ▲ 1.0 ml in water, ▼ 2.0 ml in ethylene glycol, and ■ 4.0 ml in ethylene glycol).

Since this experiment involved the acquisition of data sets that were distinguishable by the volume of particles, the depth, and the liquid used, the following synopsis of the acquisition procedure is presented: 1) the cylindrical tank was filled with 15,000 ml of liquid; 2) a predetermined volume of spherical particles was added; 3) the depth of the particle/liquid suspension was selected; 4) laser light was allowed to be incident normally upon the air/liquid interface; 5) detector readings (v) were recorded as a function of R; 6) the depth was changed, signals recorded, etc.; and 7) additional particles were added and steps 3-6 were repeated. In all cases, data were taken until the photomultiplier tube (PMT) dark level voltage was reached.

The specific values of quantities used in this experiment are presented in Table 1. The ethylene glycol index of refraction was measured[6]; the indices of refraction of the water and polystyrene spheres were calculated.[7,8] The asymmetry factor g was determined from the Mie theory.

The final quantity determined was the liquid depth Z. This was calculated from a restriction that the asymmetric optical depth τ_0^* be limited to values of ½, 1, 2, 4, and 5, where

$$\tau_0^* = (1-g)\tau_0 = (1-g)\eta C Z \tag{9}$$

Table 1 Experimental parameters for the 0.261 μm polystyrene particles[a]

	Water		Ethylene glycol	
n_{rel}	1.1969		1.083	
x	1.7230		1.9075	
1-g	0.47		0.38	
C, cm^{-1}	9183		1996	
Volume of polystyrene, cm^3	0.5	1.0	2.0	4.0
τ_0^*	Z, cm			
0.5	3.48	—	4.95	2.48
1.0	6.95	3.48	9.90	4.96
2.0	13.91	6.96	19.80	9.91
4.0	27.80	13.91	39.60	19.83
5.0	34.76	17.39	—	24.79
10.0	—	34.78	—	—

[a] $\lambda=0.6328$ μm, $\eta_w=1.3313$ (calc), $\eta_L=1.5935$ (calc), and $\eta_{eq}=1.472\pm0.0097$ (meas).

Equation (9) was used under the assumption that $\tau_0 > \tau_{liq}$ (the resulting error was less than 5%).

The detector probe was moved radially and detector signal levels were recorded. These outputs are proportional to the scattered energy transmitted to the PMT via the detector probe. In addition, the incident radiant energy power level was measured periodically.

Data Reduction and Results

The incident radiation was scattered anisotropically in the particles/liquid suspension, dependent upon the effective optical thickness. Raw data (detector output voltage v and radial distances R) were recorded for various depths, liquid type, and dispensed volumes of particles. A data reduction program[1] transformed this raw data into a nondimensional scattered intensity after processing the input parameters of the incident laser beam power level (PL), PMT dark level voltage, effective laser beam radius R_b and acceptance angle, and probe barrel radius. The program output is the following nondimensional intensity as a function of radial optical thickness τ_R^* with the optical depth τ_0^* as a parameter (see Ref. 9 for a derivation):

$$\left(\frac{R}{R_b}\right)^2 \frac{I}{I_b} = 538.86 v R_b^2 10^{-6}/\text{PL}$$

Figures 1 and 2 illustrate the program output. The heavy lines are the theoretical results.[10] These figures point out that the experimentally determined radial optical thicknesses are larger than the corresponding theoretical values. Even so, the appropriate trends are exhibited by the experimental data in terms of slope and shape as a function of τ_R^*, regardless of the liquid carrier. In addition, the relationship of the data with respect to optical depth is in general agreement with theory. Figure 3 is a typical comparison of the program output for both liquids for a given optical depth. Table 2 presents a listing of peak magnitudes of the theoretical and experimental quantities, $(R/R_b)^2$ (I/I_b) and τ_R^*, for each τ_0^* curve. Notice that there is fair agreement for all of the data presented.

It is difficult to directly compare the finite depth data presented in Ref. 4 (for a polydispersion) and the finite depth portions of these data (for a monodispersion). In general, a glance at the figures of Ref. 4 and those of this Note would indicate that the present data are somewhat more consistent (less scattered) than those of Ref. 4. This may be due to the size and relative error in computing the asymmetry factor. That is, in Ref. 4 the quantity (1-g) is on the order of 0.1, while in this presentation (1-g) is on the order of 0.4. Therefore, any small errors (e.g., in the calculation of the g measurements of the concentration of scatterers) would be more dramatic in the smaller (1-g) data.

Table 2 Comparison of theoretical and estimated experimental peak values of $(R/R_b)^2\ (I/I_b)$ and τ_R^*

τ_0^*	Theory	Experiment: Water	Experiment: Ethylene glycol
	$(R/R_b)^2\,(I/I_b)$ (at peak)		
½	$0.325\,(10^{-2})$	$0.34\,(10^{-2})$	$0.34\,(10^{-2})$
1	$0.939\,(10^{-2})$	$0.65\,(10^{-2})$	$0.95\,(10^{-2})$
2	$2.11\,(10^{-2})$	$1.95\,(10^{-2})$	$1.9\,(10^{-2})$
5	$3.7\,(10^{-2})$	$3.5\,(10^{-2})$	—
∞	4.33	—	—
	τ_R^* (for peak)		
½	0.563	0.8	0.9
1	0.922	1.1	1.2
2	1.483	1.8	1.8
5	2.35	2.6	—
∞	3.345	—	—

Conclusion

The data follow the same trends as predicted by the theory, although the experimental curves are shifted toward larger optical thicknesses. This shift appears to be independent of the medium. That is, in comparing the results found using water as the medium with those of ethylene glycol (Fig. 3), the results are virtually identical. Thus, it appears that:

1) The anisotropic nondimensional intensity distribution does depend upon the asymmetry factor g and, even though the range tested is small, $(1\text{-}g)$ appears to be a correlation parameter in anisotropic backscattering from a medium of finite depth as it was with the anisotropic backscattering from a semi-infinite medium.

2) The particle carriers produce very similar results, thus indicating the backscattered intensity is insensitive to the index of refraction of the particle carrier.

This last observation may have occurred because the range of refractive indexes used (1.33-1.47) was not large enough to produce a noticeable difference. Thus, more experimental justification is required to understand this point with certainty.

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References

[1] Tripses, J. G. and Look, D. C., "Preliminary Study of Two-Dimensional Multiple Scattering," *Letters in Heat and Mass Transfer*, Vol. 4, No. 2, 1977, pp. 129-139.

[2] Lackner, M. F. and Look, D. C., "Two-Dimensional Scattering in an Absorbing Medium," *Letters in Heat and Mass Transfer*, Vol. 6, No. 5, Sept. 1979, pp. 385-395.

[3] Garner, R., "Multiple Scattering with Absorption: A Two-Dimensional Experimental Investigation," M.S. Thesis, University of Missouri-Rolla, Rolla, Mo., 1980.

[4] Look, D. C., "Two-Dimensional Scattering from a Medium of Finite Thickness," *Journal of Heat Transfer*, Vol. 101, Aug. 1979, pp. 556-557.

[5] Look, D. C., Nelson, H. F., and Crosbie, A. L., "Anisotropic Two-Dimensional Scattering: Comparison of Experiment with Theory," *Journal of Heat Transfer*, Vol. 103, Feb. 1981, pp. 127-134.

[6] Look, D. C., "Novel Use of Total Internal Reflection," Mechanical Engineering Dept., University of Missouri-Rolla, Rolla, Mo., Rept. MAE-TM-1, Feb. 1981.

[7] Maron, S. H., Pierce, P. E., and Ulevitch, I. N., "Determination of Latex Particle Size by Light Scattering-IV," *Journal of Colloid Science*, Vol. 18, 1963, pp. 470-482.

[8] Delzelic, C. and Kratohvil, J. P., "Determination of Particle Size of Polystyrene Latexes by Light Scattering," *Journal of Colloid Science*, Vol. 16, 1961, pp. 561-580.

[9] Look, D. C., Nelson, H. F., Crosbie, A. L., and Dougherty, R. L., "Two-Dimensional Multiple Scattering: Comparison of Theory with Experiment," *Journal of Heat Transfer*, Vol. 100, Aug. 1978, pp. 480-485.

[10] Crosbie, A. L. and Dougherty, R. L., "Two-Dimensional Isotropic Scattering in a Finite Thick Cylindrical Medium Exposed to a Laser Beam," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 27, Feb. 1982, pp. 149-183.